EVALUATING THE EFFICIENCY OF QUEUING SYSTEM

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Abstract: A queue is formed when the demand for service is greater than supply (i.e. the arrival or birth is greater than departure or death). Queuing theory permits the derivation and calculation of several performance measures including the average waiting time in the queue or system, the expected number waiting or receiving service and the probability of encountering the system in certain states, such as empty, full or having to wait a certain time to be served. An example of where queue can be formed is the banking sector. The data used for this project is a primary source collected through direct observation from First Bank of Nigplc Samuel Falodun branch, Akure, for a period of ten (10) working days between the hours of 8am and 12noon and was recorded based on the arrival pattern and service pattern of customers. The M│M│C model was used where c is the number of server i.e. 2. The analytical formulas of queue theory were used in the estimation of the parameters. The underlying assumption of queuing theory ensures the arrival and service rates are Poisson and exponentially distributed respectively. The results obtained from the estimation of parameters showed that service rate μ = 33/hr is less than the arrival rate λ = 62/hr, and the traffic intensity ρ = 0.94, i.e. the probability that the servers are idle is 0.06 since ρ< 1 it implies that the service rate μ utilizes 94% of the system and remains idle for 6% of the time. If there is a cost incurred for every time spent, then by adding one more server, the cost and customer waiting times will be reduced.

Keywords: Customers, queuing system, efficiency, Poisson process

Introduction

Queuing theory is the mathematical study of waiting lines, or queues. The theory enables mathematical analysis of several related processes, including arriving at the (back of the) queue, waiting in the queue (essentially a storage process), and being served at the front of the queue. It can also be said to be a collection of mathematical models of various queuing systems. It is used extensively to analyze production and service processes exhibiting random variability in market demand. It also provides the technique for maximizing capacity to meet the demand so that waiting time is reduced drastically.

The theory permits the derivation and calculation of several performance measures including the average waiting time in the queue or the system, the expected number waiting or receiving service, and the probability of encountering the system in certain states, such as empty, full, having an available server or having to wait a certain time to be served.

Queuing theory has applications in diverse fields including banks, telecommunications, traffic engineering, computing, shops, offices and hospitals.

A bank is a corporation (that is a group of persons who accepts money on current account on demand and collects cheques from customers (Paget, 1996). The banking sector in Nigeria has witnessed significant reforms in recent times all in an effort to maximize profit, reduce cost and satisfy customers optimally in the most generally acceptable international standard. Despite these entire sterling efforts, one phenomenon remains inevitable: "queue". It is a common practice to see a very long waiting line (Queues) of customers to be serviced within the banking hall and several other places like Automated Teller Machine (ATM) points, fast food restaurants, hospitals, traffic lights, supermarket and telecommunications etc. Since the innovation of the ATM at the banks or other locations, we have witnessed a drastic reduction in the banking hall, such as the impatient customers who are time conscious and desire fast, convenient, efficient service void of human error. But even at the ATM point, the issue of queue is inevitable since no one wants to wait.

A possible cause of this problem is long waiting lines, perhaps applying queuing theory to investigate this study could help resolve the situation.

Many banks have Automated Teller Machines installed in their bank premises in order to reduce queue within the banking halls. This study therefore evaluates the efficiency of these ATMs in reducing queue in banking sector.

Materials and Methods

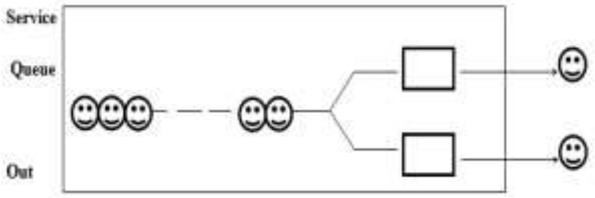
Deacon and Sonstelie (1985) evaluated customers' time value of waiting based on a survey on gasoline purchases. They found that although surveys are useful to uncover the behavioural process by which waiting affects customer behaviour and the factors that mediate this effect; they suffer from some disadvantages.

Mandelbaum and Zeltyn (2005) used analytical queuing models with customer impatience to explain nonlinear relationships between waiting time and customer abandonment. They found that, in the context of call-center outsourcing, the common use of service level agreements based on delay thresholds at the upper-tail of the distribution (e.g. 95% of the customers wait less than 2 min) was consistent with non-linear effects of waiting on customer behaviour.

Campbell and Frei (2009) studied multiple branches of bank, provided empirical evidence that teller waiting times affect customer satisfaction and retention. Their study revealed significant heterogeneity in customer sensitivity to waiting time, some of which could be explained through demographics and the intensity of competition faced by the branch.

Eze and Odunukwe (2013) examined the application of queuing models to customers management in the banking system using United Bank for Africa, Okpara Avenue Branch Enugu, as a case study. The results obtained from the study showed that the arrival pattern follows a Poisson distribution and that the service pattern follows an exponential distribution. The study recommended that the Bank management should increase the number of servers to three so as to help reduce the time customers spend on queue and also reduce cost incurred from waiting.

The queuing model considered in this work is M/M/2. It describes a system where arrivals form a single queue and are governed by a Poisson process, there are 2 servers and service times are exponentially distributed as shown below.



Basically, the data used for this study were obtained from primary source, which was limited to First Bank Nig. Plc., Akure, The method of data collection was through direct observation. A sport watch, a pen and a notepad were requirements needed for the recording of relevant information such as: number of customers, the arrival times of customers, service starts time, and service departure time. The observation was made during the working hours of (8am-12pm) for a period of (10 days).

The computational formulas for estimating the parameters of this queuing system are as follow:

Arrival rate $\lambda = \frac{\text{Total number of arrivals}}{\text{Total time}}$,

Service rate $\mu = \frac{\text{Number of customers being served}}{\text{Total time of service}}$

Traffic intensity (ρ) which is the proportion of the average arrival rate to the average service rate is obtained by:

$$\rho = \frac{\lambda}{c\mu},$$

where, λ is the mean arrival rate, μ is the mean service rate, and c is the number of servers (i.e. 2).

Also, probability of zero customers at the ATM (P_0) is obtained by

$$\text{Probability } (n=0) = P_0 = \frac{c!(1-\rho)}{(\rho c)^c + c!(1-\rho)\left[\sum_{n=0}^{c-1}\frac{1}{n!}(\rho c)^n\right]},$$

Probability that an arriving customer has to wait in queue (P_n) is

$$P_n = \frac{(\rho c)^c P_0}{c!(1-\rho)},$$

Average number of expected customers in the system (L_s) can be obtained by

$$L_s = \frac{\lambda}{(c\mu-\lambda)}$$

Average number of expected customers in the queue (L_q) can be obtained by

$$L_q = \frac{\lambda^2}{c\mu(c\mu-\lambda)}$$

Average expected time a customer spends in the system (W_s) can be computed by

$$W_s = \frac{1}{(c\mu-\lambda)}, \text{ and}$$

Average Waiting time of customers in the queue (W_q)

$$W_q = \frac{\lambda}{c\mu(c\mu-\lambda)}.$$

Results and Discussion

The arrival and service rates are obtained as λ = 62 and μ = 33, respectively. The traffic intensity or utilization factor ρ is obtained to be 0.94 while the idle ratio is 0.06. The probability of zero customer, P_0 is 0.03 and the probability of queuing on arrival, P_n is 0.88. The following were also computed: Average number of expected customers in the system, L_s = 16 customers, average number of expected customers in the queue, L_q = 14 customers.

Table 1: Performance measures of queuing system with increasing number of servers

Performance measures	Number of servers			
	2	3	4	5
ρ	0.93939	0.626263	0.469697	0.375758
L_q	14.1056	0.651968	0.128879	0.028585
L_s	15.9844	2.530756	2.007667	1.907373
W_q	0.22751	0.010516	0.002079	0.000461
W_s	0.25781	0.040819	0.032382	0.030764
P_0	0.03125	0.131559	0.148633	0.15196
Cost-based on waiting	1610.56	365.1968	412.8879	502.8585
Cost-based on system	1798.44	553.0756	600.7667	690.7373

Considering the analytical solution, the capacity of the system under study is 2503 customers out of which 2472 customers were patient to receive service while 31 customers reneged and the arrival rate is 62 customers per hour while the service rate is 33 customers per hour. The arrival rate can be seen to be greater than service rate, this implies that customers have to queue up; though the queue will not be long, probability that the servers are idle is 0.06 which shows that the servers will be 6% idle and 94% busy.

Considering cost evaluation, the average cost per day for waiting (assuming cost incurred for time is ₦100 with c = 2) is **₦11065.76** and for c = 3, the average cost per day for waiting is **₦521.59**, which means there is a saving in the expected cost of **₦10544.17**.

From this study, it can be recommended that at least a server should be added to the existing two. Adding one more server will drastically reduce the time customers spend on a queue and as well help to reduce the cost incurred from waiting.

Conflict of Interest

Author declares that there is no conflict of interest.

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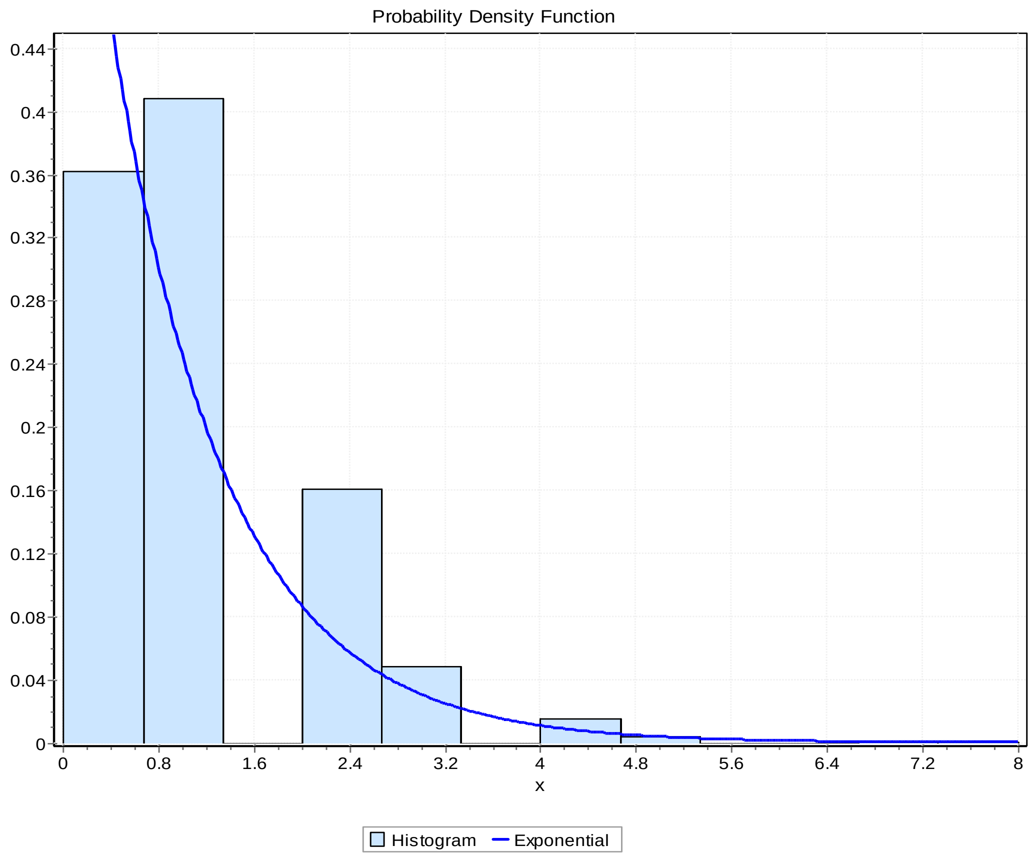
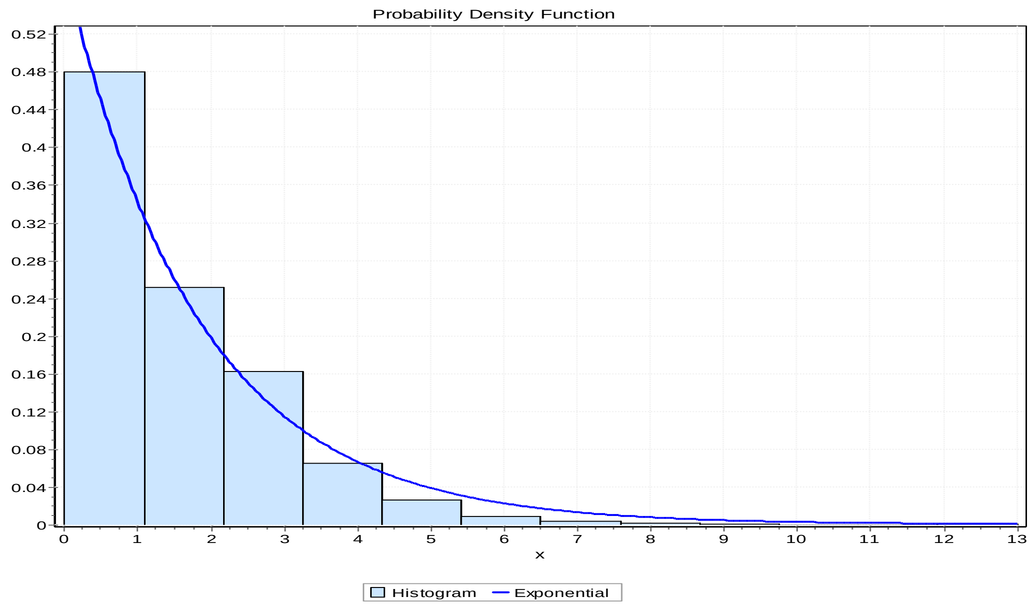
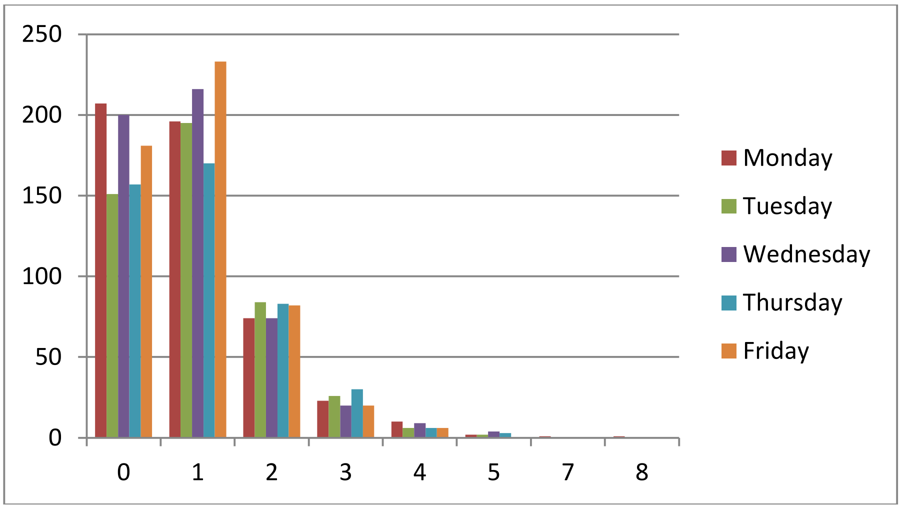
APPENDIX: SOME DESCRIPTIVE GRAPHICAL RESULTS



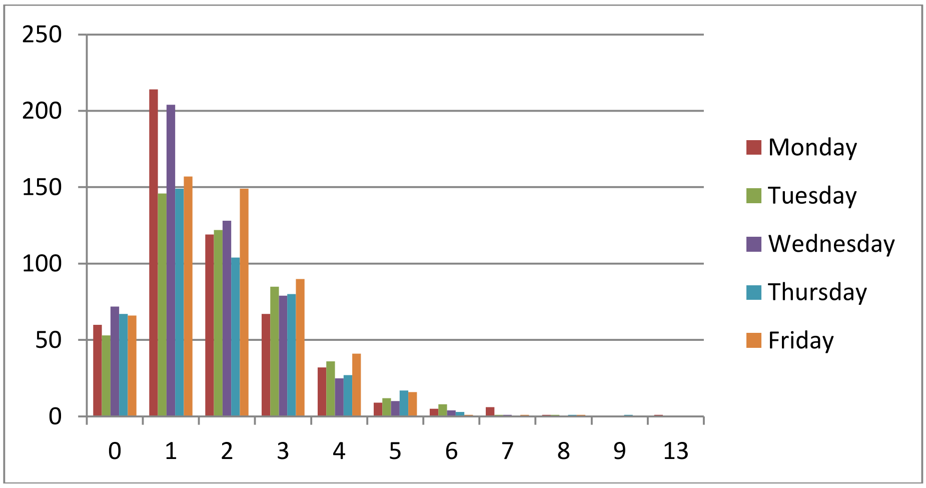
Appendix 1: Fit of inter-arrival time



Appendix 2: Fit of service times



Appendix 3: Histogram of arrival rate



Appendix 4: Histogram of service rate